PROGRAM & ABSTRACTS

Quy Nhon, June 4, 2022

PROGRAM

Offline at Quy Nhon Universtiy (**Meeting Room 3** at the 5th Floor)
and Online via Zoom
Link: `https://us02web.zoom.us/wc/join/2028299761?wpk=wcpkee430c88acefabb2893dcf988de9b109`
Meeting ID: **202 829 9761** Passcode: **12345**

Time	**Speaker**	**Title**
8am-8:45am	Rajarama Bhat	C^*-extreme points of entanglement breaking maps
8:45am-9:10am	Ngoc Thieu Vo	On separant matrices of algebraic differential equations and formal power series solutions
9:10am-9:35am	Jose Franco	Generalized In-Betweenness of Positive Matrices
9:35am-9h:50am	**Break**	
9:50am-10:15am	Huajun Huang	Multiplicative trace and spectrum preserving maps on matrix spaces
10:15am-10:40am	Anh Vu Le	Lie Algebras, Lie Groups: Some New Results about their Classification and Representation
10:40am-11:05am	Tin-Yau Tam	Symplectic analog of Ballantine's matrix product theorem
11:05am-11:30am	Trung Hoa Dinh	Geometric properties of matrix means in different distance functions
11:30am-1:30pm	**Break**	
1:30pm-2:15pm	Takeaki Yamazaki	The induced Aluthge transformations
2:15pm-2:40pm	Trung Dung Vuong	The alpha-z weighted right mean and alpha-z fidelity
2:40pm-3:05pm	Thanh Hieu Le and Nguyen Thi Ngan	The SDC problem of collections of real symmetric matrices
3:05pm-3:30pm	Hiroyuki Osaka	On a class of k-entanglement witnesses
3:30pm-3:45pm	**Break**	
3:45pm-4:10pm	Seung-Hyeok Kye	Mapping cones between matrix algebras in quantum information theory
4:10pm-4:35pm	Minh Toan Ho and The Khoi Vu	Metric properties of functionals of quantum fidelities
4:35pm-5pm	Kijti Rodtes	Matrices over commutative rings as sum of higher powers
5pm-5:25pm	Ha Văn Hieu	Classification of Simply Connected Real Lie groups whose nontrivial Coadjoint orbits are all of codimension not exceed 2
5:25pm	**CLOSING**	
7pm-11pm	**PARTY**	

ABSTRACTS

PLENARY PRESENTATIONS

C^*-extreme points of entanglement breaking maps

B V Rajarama Bhat, Indian Statistical Institute, India Email: bvrajaramabhat@gmail.com

Abstract. We study the C^*-convexity structure of unital entanglement breaking maps on matrix algebras. By establishing a Radon-Nikodym type theorem we give a complete description of their C^*-extreme points. In particular, it is seen that a unital entanglement breaking map from M_m to M_n is C^*-extreme if and only if it has Choi rank equal to n. This is based on a joint work with R. Devendra, N. Mallick and K. Sumesh.

The induced Aluthge transformations

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Abstract. Let $B(\mathcal{H})$ be the C^*-algebra of all bounded linear operators on a complex Hilbert space $\mathcal{H}$. Let $T = U|T|$ is the polar decomposition of $T \in B(\mathcal{H})$. The Aluthge transformation $\Delta(T) := |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$ is a well-known mapping which was firstly introduced by A. Aluthge [1]. It has very nice properties, for example, (i) $\sigma(T) = \sigma(\Delta(T))$, (ii) if T is a n–by–n matrix, then a matrices sequence $\{\Delta^n(T)\}_{n=0}^{\infty}$ converges to a normal matrix [2, 3], and T has a non-trivial invariant subspace if and only if $\Delta(T)$ does so [4].

By the way, Aluthge transformation can be considered as a geometric mean of left and right multiplication operators in the case $\mathcal{H} = \mathbb{C}^n$. For an operator $T \in B(\mathbb{C}^n)$, left and right multiplication operators on $B(\mathbb{C}^n)$ are defined as follows:

$$\mathbb{L}_T X := TX \quad \text{and} \quad \mathbb{R}_T X := XT \quad \text{for } T \in B(\mathbb{C}^n).$$

Obviously, if $T \geq 0$, the $\mathbb{L}_T$ and $\mathbb{R}_T$ are positive operators on $B(\mathbb{C}^n)$ under the inner product $\langle A, B\rangle = \mathrm{tr}(AB^*)$, and it is easy to see that $f(\mathbb{L}_T) = \mathbb{L}_{f(T)}$ and $f(\mathbb{R}_T) = \mathbb{R}_{f(T)}$ hold for any analytic function f defined on $\sigma(T)$. Moreover $\mathbb{L}_T$ and $\mathbb{R}_S$ are commuting, i.e.,

$$\mathbb{L}_T\mathbb{R}_S X = \mathbb{R}_S\mathbb{L}_T X = TXS.$$

Then the Aluthge transformation can be represented by the following form:

$$\Delta(T) = \mathbb{L}_{|T|}^{\frac{1}{2}}\mathbb{R}_{|T|}^{\frac{1}{2}}U,$$

i.e., the geometric mean of $\mathbb{L}_{|T|}$ and $\mathbb{R}_{|T|}$ is appeared.

In this talk, we shall extend the Aluthge transformation by considering other operator means. To introduce extensions, we shall divide matrices and operators cases.

This talk is based on [5].

References

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[2] T. Ando and T. Yamazaki, *The iterated Aluthge transformations of a 2–by–2 matrix converge*, Linear Algebra Appl., **375** (2003) 299–309.

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[5] T. Yamazaki, *The induced Aluthge transformationations* Linear Algebra Appl., **628** (2021), 1–28.

CONTRIBUTED PRESENTATIONS

On separant matrices of algebraic differential equations and formal power series solutions

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Abstract. We propose a computational method to determine when a truncated power series can be extended to a formal power series solution for an algebraic ordinary differential equation (AODE). When the existence is confirmed, we present the algebraic structure of the set of all formal power series solutions. The method is based on the generalized separant matrix of an AODE.

Generalized In-Betweenness of Positive Matrices

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Abstract. A matrix mean σ is said to satisfy the *in-betweenness property* with respect to the metric d if for any pair of positive definite matrices A and B,

$$d(A, A\sigma B) \leq d(A, B).$$

In this talk, we study the *geodesic in-betweenness property* for k-tuples of positive definite matrices with respect to any metric. Moreover, we show that in spaces of non-positive curvature, geodesic in-betweenness implies in-betweenness in the case of two matrices. We then show some examples of metrics and means for which this property is satisfied. We close the talk with some open problems relating to generalized in-betweenness.

Multiplicative trace and spectrum preserving maps on matrix spaces

Huajun Huang, Auburn University, USA
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Abstract. Preserver problems study maps between spaces or sets that leave certain sets, functions, and relations invariant. The study of spectrum preserving maps has attracted researchers in matrix and operator theorists for long. Trace preserving maps are easier to verify and they include spectrum preservers. In this talk, we describe multiplicative trace and spectrum preserving maps on the spaces of square, symmetric, Hermitian, positive semidefinite, diagonal, nonnegative, and stochastic matrices. The talk is based on joint works with Ming-Cheng Tsai.

Lie Algebras, Lie Groups: Some New Results about their Classification and Representation

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Abstract. In the first part of the talk, we will introduce an overview about Lie algebras and some new results about the problem of classification of them. The second part of the talk is devoted to the discussion of some new results about representations of Lie Groups.

Symplectic analog of Ballantine's matrix product theorem

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Abstract. In the late 1960 Ballantine showed that every matrix with positive determinant is a product of five positive definite matrices.

We consider the complex symplectic group $\mathrm{Sp}(2n, \mathbb{C})$:

$$\mathrm{Sp}(2n, \mathbb{C}) = \left\{ A \in \mathrm{GL}(2n, \mathbb{C}) : A^\top J_n A = J_n \right\},$$

where

$$J_n = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}.$$

The symplectic group is a classical group defined as the set of linear transformations of a $2n$-dimensional vector space over $\mathbb{C}$, which preserve the non-degenerate skew-symmetric bilinear form that is defined by J_n. We show that every symplectic matrix is a product of five positive definite symplectic matrices. We also show that five is the best in the sense that there are symplectic matrices which are not product of less.

This is a joint work with Daryl Q. Granario, De La Salle University, Philippines.

Geometric properties of matrix means in different distance functions

Trung Hoa Dinh, Troy University, USA
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Abstract. In this talk we introduce several new quantum divergences on the set of positive definite matrices. We also study some geometric properties of matrix means in different distance functions.

α-z-weighted right mean and α-z-fidelity

Trung Dung Vuong, Quy Nhon University, Viet Nam
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Abstract. In this talk we present some properties of the α-z weighted right mean, the solution to the least square problem of α-z Wasserstein distance. We are able to show that it is a multivariate Lie-Trotter mean. In addition, we measure the distance between two quantum orbits using the α-z fidelity. This is a joint work with Trung Hoa Dinh and Cong Trinh Le.

The SDC problem of collections of real symmetric matrices

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Abstract. Let $C_1, C_2, \dots, C_m$ be $n \times n$ real symmetric matrices. The SDC (simultaneous diagonalization via congruence) problem of $C_1, C_2, \dots, C_m$ asks:

1. whether there is a nonsingular matrix P such that $P^T C_i P$ are all diagonal?
2. if exists, how to find such a matrix P?

Our recent study [1] obtained a necessary and sufficient condition for the existence of P and proposed an algorithm for finding P for a nonsingular collection, i.e., at least one of the matrices is nonsingular.

For singular collection, we show inductively that if $n-1$ matrices $C_1, C_2, \dots, C_{m-1}$ are SDC, then there exist a vector $\mu = (\mu_1, \mu_2, \dots, \mu_{m-2}, 1) \in \mathbb{R}^{m-1}$ and a nonsingular matrix Q such that:

- the matrices C_i are transformed to $\bar{C}_i$ as follows

$$\begin{aligned}
\bar{C}_1 &:= Q^T C_1 Q = \operatorname{diag}(C_1^{(1)}, 0_r), \\
\bar{C}_2 &:= Q^T(\mu_1 C_1 + C_2)Q = \operatorname{diag}(C_2^{(1)}, 0_r), \\
\bar{C}_3 &:= Q^T(\mu_2(\mu_1 C_1 + C_2) + C_3)Q = \operatorname{diag}(C_3^{(1)}, 0_r), \\
&\dots \\
\bar{C}_{m-1} &:= Q^T(\mu_{m-2}(\dots\mu_3(\mu_2(\mu_1 C_1 + C_2) + C_3) + C_4) + \dots + C_{m-2}) + C_{m-1})Q \\
&\quad = \operatorname{diag}(C_{m-1}^{(1)}, 0_r),
\end{aligned} \tag{1}$$

  such that the sub-matrices $C_i^{(1)}, i = 1, 2, \dots, m-1$, are all diagonal and of the same size, suppose $p \times p$; $C_{m-1}^{(1)}$ is nonsingular; and

$$\bar{C}_m := Q^T C_m Q = \begin{pmatrix} C_m^{(1)} & 0_{p\times s} & C_m^{(2)} \\ 0_{s\times p} & C_m^{(3)} & 0_{s\times(r-s)} \\ (C_m^{(2)})^T & 0_{(r-s)\times s} & 0_{r-s} \end{pmatrix}, s \le r, \tag{2}$$

  such that C_1^m is a $p \times p$ symmetric matrix, $C_m^{(3)}$ is nonsingular diagonal of size $s \times s$; $C_m^{(2)}$ is either a $p \times (r-s)$ matrix if $s < r$ or does not exist if $s = r$.

- $C_1, \dots, C_m$ are SDC if and only if $C_1^{(1)}, \dots, C_m^{(1)}$ are SDC and $C_m^{(2)}$ either is zero or does not exist. It should be noted that $C_1^{(1)}, \dots, C_m^{(1)}$ form a nonsingular collection.

In this talk, we present the technique to find the vector μ and the matrix Q above.

Keywords: real symmetric matrices, simultaneous diagonalization via congruence, nonsingular collection, singular collection.

[1] Ngan Nguyen Thi,Van Bong Nguyen, Thanh Hieu Le and Ruey Lin Shue : On simultaneous diagonalization via congruence of real symmetric matrices. https://arxiv.org/abs/2004.06360, https://arxiv.org/abs/arXiv:2004.06360.

On a class of k-entanglement witnesses

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Abstract. Recently, Yang et al. showed that each 2-positive map acting from $\mathcal{M}_3(\mathbb{C})$ into itself is decomposable. It is equivalent to the statement that each PPT state on $\mathbb{C}^3 \otimes \mathbb{C}^3$ has Schmidt number at most 2. It is a generalization of Perez-Horodecki criterion which states that each PPT state on $\mathbb{C}^2 \otimes \mathbb{C}^2$ or $\mathbb{C}^2 \otimes \mathbb{C}^3$ has Schmidt rank 1 i.e. is separable. Natural question arises whether the result of Yang et al. stays true for PPT states on $\mathbb{C}^3 \otimes \mathbb{C}^4$. This question can be considered also in higher dimensions. We construct a positive maps which is suspected for being a counterexample. More generally, we provide a class of positive maps between matrix algebras whose k-positivity properties can be easily controlled.
This is mainly based on joint work with Tomasz Mlynik and Marcin Marciniak (arXiv:2104.14058, 2021).

Mapping cones between matrix algebras in quantum information theory

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Abstract. A mapping cone is a closed convex cone of positive linear maps which is closed under composition by completely positive maps from both sides. One-sided mapping cones are defined in similar ways. We characterize those notions in terms of tensor products of linear maps. To do this, we exhibit an identity which connects compositions and tensor products of linear maps through Choi matrices. As applications, we give several equivalent statements equivalent to the PPT square conjecture. We also reinterpret various mapping cones arising from quantum information theory, and show that the definition of Choi matrix is independent of the choice of matrix units to some extent. This talk is based on two papers, [Girard, Kye and Størmer, Linear Alg. Appl. 608(2921), 248-269, arXiv 2002.09614] and [Kye, arXiv 2204.02516].

Matrices over commutative rings as sum of higher powers

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Abstract. It is well known in number theory that every natural number can be represented as the sum of four integer squares: "*Lagrange's four-square theorem*" or "*Bachet's conjecture*". There is also a corresponding problem asking whether each natural number k has an associated positive integer $g(k)$ (depending only k) such that every natural number is the sum of at most $g(k)$ natural numbers raised to the power k, which is known as "*Waring's problems*". This problem can be generalized naturally to deal with the rings of matrices; "*Waring's problem for matrices over rings*"

1. Given a matrix $M \in M_n(R)$, can M be written as a sum of k-th powers in $M_n(R)$?
2. Given a ring R, can every $M \in M_n(R)$ be written as sum of k-th powers in $M_n(R)$?
3. Find the least positive integer $g(n,k,R,M)$ such that a given matrix $M \in M_n(R)$ can be written as a sum of $g(n,k,R,M)$ k-th powers in $M_n(R)$.
4. Find the least positive integer $g(n,k,R)$ such that every matrix $M \in M_n(R)$ which can be written as a sum of $g(n,k,R)$ k-th powers in $M_n(R)$.

Here, the ring R will always mean an associative ring with identity and $M_n(R)$ denotes the set of all square matrices of size n over the ring R. In this talk we will discuss the recent development of the questions (1) and (2) for matrices over commutative rings as sum of 9,10,11,12,13,14,15,16 th powers.

The speaker would like to thank department of mathematics and faculty of science, Naresuan University, Thailand, for financial support.

Keywords: Sum of powers, Waring's problem, Matrices over commutative rings.

References

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Metric properties of functionals of quantum fidelities

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Abstract. In this talk, we give positive answers to some questions about the metric property of functionals of alternative quantum fidelities on the space of mixed quantum states and discuss some extensions to the space of psd matrices.

Classification of Simply Connected Real Lie groups whose nontrivial Coadjoint orbits are all of codimension not exceed 2

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Abstract. We use techniques in Matrix theory to give a complete classification of subclass of simply connected real Lie groups whose nontrivial coadjoint orbits are all of codimension not exceed 2. Such a Lie group belongs to a well-known class, called the class of MD-groups. By definition, an MD-group is a solvable real Lie group such that its nontrivial coadjoint orbits have the same dimension. The Lie algebra of an MD-group is called an MD-algebra. Some interest properties of MD-algebras will be investigated as well.

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